PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Sea-Level Observations and their Secular Variation

J. R. Rossiter

Phil. Trans. R. Soc. Lond. A 1972 **272**, 131-139
doi: 10.1098/rsta.1972.0039

Phil. Trans. R. Soc. Lond. A. **272**, 131–139 (1972) [131]
Printed in Great Britain

Sea-level observations and their secular variation

By J. R. Rossiter
Institute of Coastal Oceanography and Tides,
The Observatory, Bidston, Birkenhead L43 7RA, Cheshire

The analysis of continuous sea-level records at the coastal stations is direct and should be one of the most reliable means of assessing secular coastal movement in the vertical. However, limitations in the traditional techniques used to obtain raw data, and the existence of sea-level variations due to oceanographic and meteorological phenomena require more than a simple univariate regression analysis for best results.

Some examples from situations in western Europe will be given which have a bearing upon the subsidence of southeast England.

The earliest recorded systematic observations of high-water heights in the United Kingdom were by Samuel Colepress at Plymouth and Captain Samuel Sturmy at Bristol, reported in 1668. It was not until the beginning of the nineteenth century that similar observations appear to have been made in the Thames, and even then little attention was paid to low water heights.

The use of reasonably precise sea-level observations as an indicator of the relative movement of sea and land in the vertical would not therefore seem able to throw light upon the situation existing before the last century. In practice, the position is much worse than this since permanent continuous tidal recordings of a suitable quality do not exist before 1860, and then only for very few stations. Figure 1 shows the modern network of first-class tidal stations in the United Kingdom, and figure 2 indicates the span of annual mean sea-level data associated with these and other stations, past and present.

It will be seen that the coastline south of the Humber and east of the Solent can only provide three series containing as many as 20 years of reasonably complete data (Felixstowe, Southend and Sheerness) of which the first, regrettably, was discontinued as a recording station in 1950. The oldest series, that for Sheerness, is intermittent and its sponsors, the Hydrographic Department, consider the data to be partially suspect.

Thus our basic data for a study of the relative movement of land and sea in southeast England are not conspicuously plentiful.

The concept of mean sea level

For geodesists unfamiliar with the fundamentals of physical oceanography the ‘mean’ sea surface has long appeared an attractive equipotential reference level for the measurement of terrestrial elevations. In line with this thinking the first and second Ordnance Surveys used observations of mean sea level at Liverpool and Newlyn in 1844 and in 1915–21 respectively to determine ordnance datum. For their part, oceanographers have tended to refer sea-level observations to ordnance datum, a fixed base on dry land. It is therefore not surprising that considerable confusion and misunderstanding has arisen at this interface of disciplines and elements, the coastline, which have only been systematically clarified during the last 20 years.

Observed mean sea level at a given station, i.e. the mean of an observed time series over a given span of time, will invariably exhibit variations, the variance of which decreases as the averaging span increases. These variations arise from oceanographical, meteorological and terrestrial sources; instrumental and computational noise contributions must also be included for a complete picture. At best they form an embroidery to, at worst they mask the secular variation of sea level relative to the land, and must be identified and assessed in this context.

Figure 1. The modern network of United Kingdom first class tidal stations.

Instrumental and computational noise

The standard, graphically recording tide gauge station has been developed very little since Palmer's days (1831) until quite recently. Characteristic errors are both quasi-periodic (mechanical friction, hygro-expansion and inadequate calibration of paper charts, stilling-well draw-down, etc.) and purely random (datum instability, inadequate operation, etc.).

The quasi-periodic errors will be largely (but not wholly) eliminated by suitable averaging processes. Of the more random errors, datum changes due to inadequate operation are distressingly frequent, and have been established as the largest source of error.

Individual readings from a tide gauge graph cannot be obtained to better than 1.5 cm (0.05 ft); random errors of this magnitude may be expected to give standard errors of 0.6 and 0.15 mm (0.0019 and 0.0005 ft) in monthly and annual means respectively. In practice, however, and despite elaborate computerized quality control, much larger errors can contaminate the raw data.

Many computational schemes exist for producing mean values from the raw data; they range from straightforward averaging of 24 h values to sophisticated numerical filters. The main function of all the schemes is to suppress the dominant stationary tidal oscillations, i.e. to act as low pass filters with a cut-off frequency at about 1 cycle/day. The daily means so produced are then subjected to simple averaging to give monthly and annual means, contributing to a further suppression of oscillations in the frequency range 1 cycle/day to 1 cycle/year.

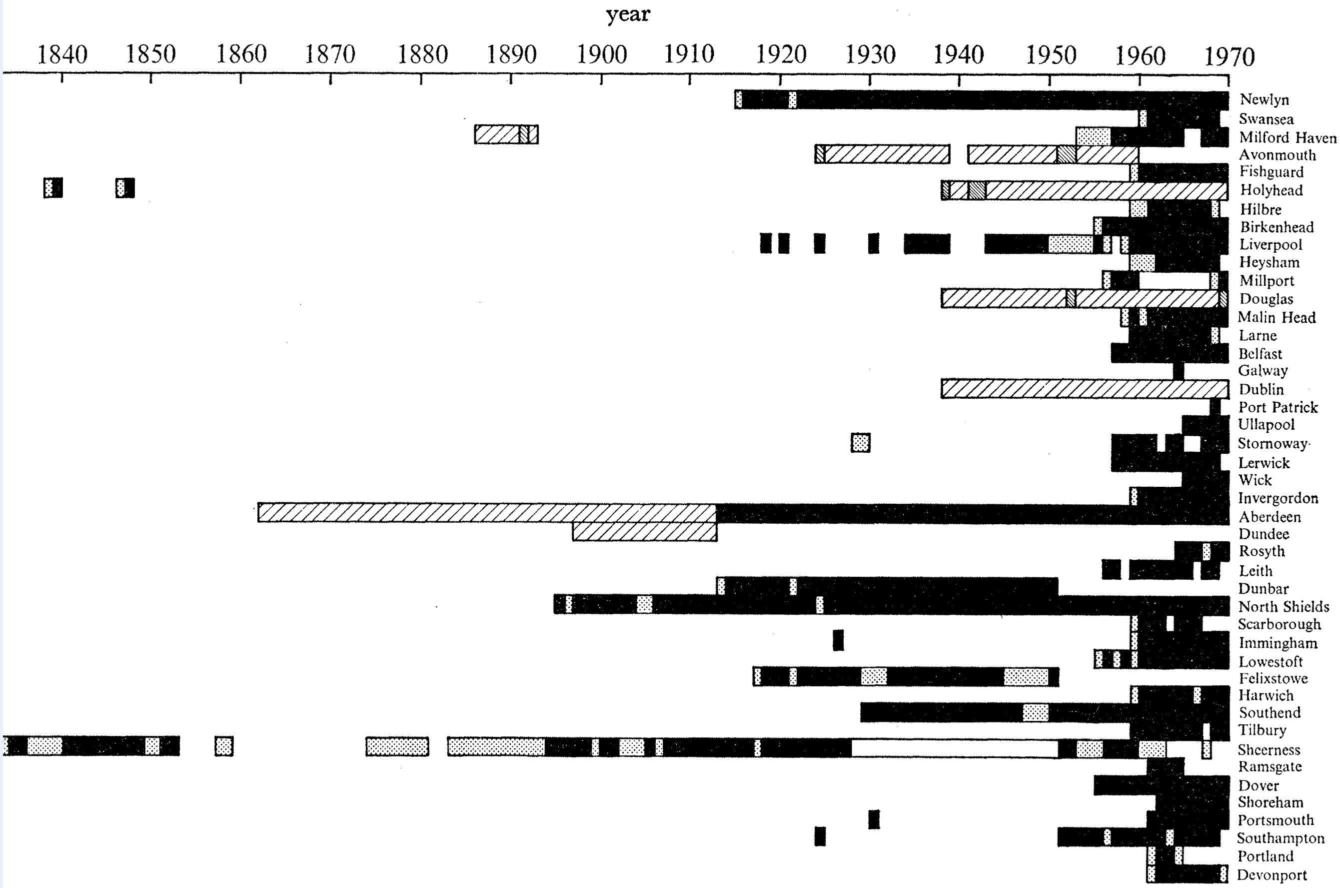

Figure 2. Mean sea-level data available from United Kingdom stations. ■, complete year of mean sea-level data; ▦, incomplete year of mean sea-level data; ▨, complete year of mean tide-level data; ▧, incomplete year of mean tide-level data.

The efficiency of the simple averaging process of 24 h heights to the day is indicated in table 1. The maximum contributions are associated with a cosine term the argument of which, in general, takes up all values between 0 and 2π. Thus for a moderate amplitude (1.5 m) of M_2, the principal lunar constituent, there will be maximum contributions to monthly and annual means of the order of 1.1 and 0.7 mm (0.003 and 0.002 ft) respectively.

In general, unless the most careful attention is paid to tide gauge maintenance and data processing, annual values of tide-free mean sea level can only be considered accurate to something like 1 mm (0.003 ft).

TABLE 1. MAXIMUM CONTRIBUTION FROM CERTAIN TIDAL CONSTITUENTS EXPRESSED AS PERCENTAGES OF THEIR AMPLITUDES, TO THE MEANS FOR (*a*) A 30-DAY MONTH AND (*b*) A 365-DAY YEAR

	M_2	N_2	K_1	O_1	M_4	M_6	Msf
(*a*)	0.055	0.209	0.267	0.401	0.058	0.060	1.55
(*b*)	0.035	0.005	0.000	0.072	0.023	0.008	1.27

M_2: principal lunar semidiurnal tide, period 12.42 h.
N_2: lunar elliptic semidiurnal tide, period 12.66 h.
K_1: lunar declinational diurnal tide, period 23.93 h.
O_1: lunar declinational diurnal tide, period 25.82 h.
M_4: lunar quarter-diurnal tide, period 6.21 h.
M_6: lunar sixth-diurnal tide, period 4.14 h.
Msf: fortnightly tide, largely arising from shallow water theory, period 14.8 days.

Annual anomalies of mean sea level

Accepting the conclusion of the preceding section, series of annual mean values of sea-level in units of 1 mm constitute the data for studying secular variations. Such a series for Newlyn (figure 3) covering the period 1916–68 immediately reveals significant annual anomalies, relative to the equally obvious trend, having a standard deviation of 37.5 mm. It is therefore clearly desirable to understand and purify the series of these anomalies, so far as possible, to obtain a best estimate of the secular variation. In the absence of a complete theoretical knowledge of the disturbing forces responsible for the anomalies, including estimates of the physical parameters involved, an empirical, time-dependent regression analysis is indicated.

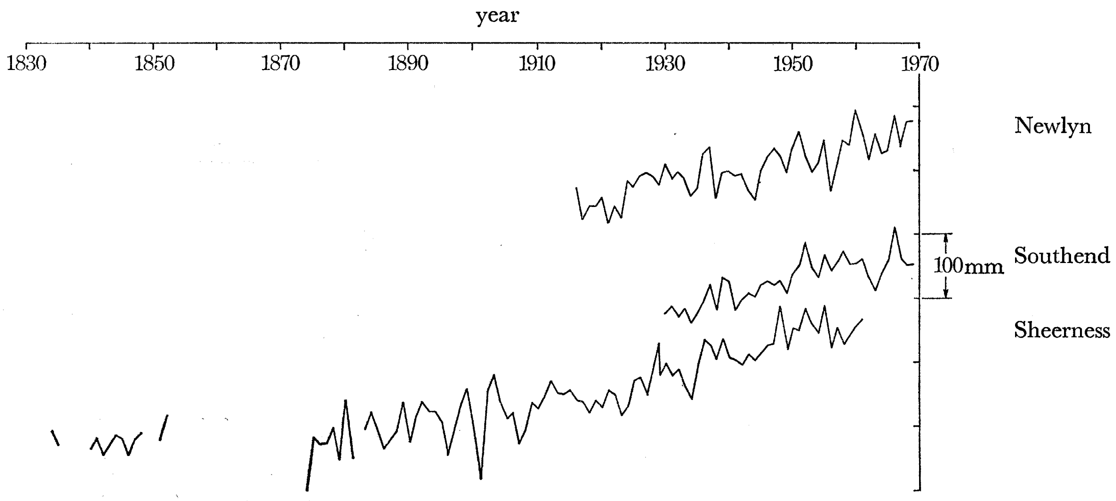

FIGURE 3. Annual values of mean sea-level at Newlyn, Sheerness and Southend.

Anomalies of oceanographic origin

The form of the sea surface is determined by the three-dimensional distribution of currents and density. Annual anomalies in these distributions must be reflected in corresponding anomalies in mean sea level. Steric changes on a seasonal scale over large oceanic areas, for example, have been successfully linked with sea-level changes. But time series of temperature and salinity in depth, comparable to the annual sea-level series, do not exist, and with one exception there seems no scope for the introduction of independent oceanographical variables into a regression analysis.

The exception is the nodal tide with a period of 18.61, the only significant tidal oscillation with a frequency less than 1 cycle/year.

Anomalies of climatological origin

The mean sea surface is in a state of equilibrium or quasi-equilibrium with both the normal air pressure and tangential surface wind stress according to the time scale considered. From a physical viewpoint, the meteorologically conditioned response of the air–sea interface is most usefully understood by considering units of an hour. Except in the case of intense, fast-travelling depressions the inverted water barometer effect, in which a change of 1 mbar (100 Pa) air pressure produces a change of 1 cm sea level, can be expected to hold without an appreciable time lag. Due to the inertia of the water and to energy losses, the much more dynamic and relatively larger response to wind stress is invariably associated with time lags of the order of 6 h in coastal waters.

Hourly anomalies of sea level can therefore be correlated quite successfully with hourly anomalies of air pressure and surface wind, and this is the basis of storm surge research and forecasting. It follows that annual means of these variables can also be correlated, and the meteorological data exist to make this possible in the form of annual mean air pressures at standard meteorological stations. A linear relationship between wind stress and surface wind is necessarily assumed.

Anomalies of terrestrial origin

Under this heading are found those anomalies of mean sea level which arise from changes in the distance between the centre of the Earth and the datum of sea-level observations, that is to say, the tide gauge bench mark. Sudden movements of the Earth's crust will appear as true annual anomalies; others, such as slow isostatic adjustments, in the secular trend. Since neither can be continuously monitored independently, they cannot be incorporated in a regression analysis.

Unless the movements are sudden and clearly exceed the norm of annual sea-level anomalies (the Chilean earthquake of 1960 moved the datum at Antofagasta by 60 mm) they can only be recognized and quantified by repeated geodetic levellings.

A mean sea-level regression analysis

From the foregoing, a regression analysis based on the equation

$$Z_Y = \sum_{p=0} a_p Y^p + \sum_{r=1} b_r B_r + c_1 \cos N + c_2 \sin N + \phi_Y \tag{1}$$

is therefore practical, where a, b, c are coefficients to be determined, Y is the year number (relative, say, to 1900), $\sum_{p=0} a_p Y^p$ represents the secular variation in the form of a polynominal of order p, B_r is the annual anomaly of mean air pressure at station r, $\sum_{r=1} b_r B_r$ represents the meteorological contributions to sea level, from both air pressure and wind stress, derived from triads of stations ($r = 3$) covering each of the sea areas over which these forces may be considered significant, $N = f(Y)$, the mean longitude of the moon's ascending node, $c_1 \cos N + c_2 \sin N$ represents the nodal tide, and ϕ_Y represents the contribution to sea level from all other sources.

The merits of a multivariate analysis of this kind were argued by the author (1967) and illustrated by details for Esbjerg. Figure 4 shows the various components of annual anomaly for the years examined. The standard deviations of the original data, of the atmospheric contributions and of the residuals are 49, 35 and 20 mm respectively, and indicate a significant correlation between the chosen meteorological variables and mean sea level. The nodal tide contribution is small and ill-defined, the amplitude being 9 ± 5 mm. The linear trend line is 1.48 ± 0.15 mm/year and the total correlation coefficient is 0.914.

In comparison, a simple univariate regression analysis of the same data for a linear trend produces an estimate of 1.21 ± 0.23 mm/year, with a total correlation coefficient of 0.529 and residuals having a standard deviation of 42 mm.

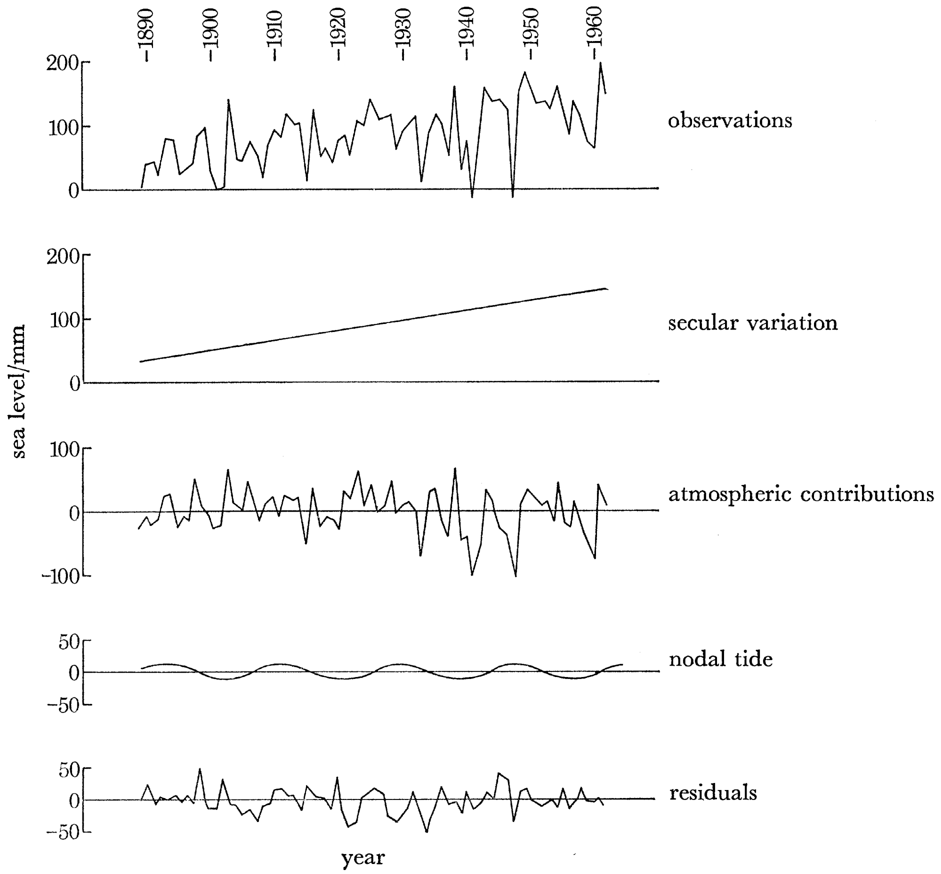

FIGURE 4. Components of sea-level variations at Esbjerg, 1889–1962

Although the two trend estimates of 1.48 and 1.21 mm/year are not significantly different, the statistics associated with the former provide greater confidence. In passing, it is interesting to note that of 97 European stations analysed, only five gave residuals with standard deviations of less than 15 mm; four of these were British stations, and the lowest values came from the only stations operated by the Ordnance Survey: Newlyn (11 mm), Felixstowe (10 mm) and Dunbar (11 mm). The inescapable conclusion is that these results are a measure of the care lavished upon the basic observations.

Studies of secular changes in British waters

Very few analyses of mean sea-level data at British stations have been made for the purpose of assessing vertical movements, possibly because of the scarcity of such data. Witting (1918) used

Aberdeen and Dundee records in his extensive studies covering the Baltic and the North Sea. Longfield (1932) drew attention to the rise of mean sea level at Felixstowe which, taken in conjunction with a similar rise at Newlyn, led Jolly (1939) to suppose a land subsidence in the south of England. Gutenburg (1941) calculated the secular rise of sea level at all available stations throughout the world to produce an estimate for the eustatic rise in sea level of 1.1 mm/year. Egedal, Disney & Rossiter (1954) also used all existing uncorrected data, but refrained from inferring the eustatic rise. Munk & Revelle (1952), however, limited their global investigations to the decades 1905–15 and 1915–25 and concluded that 1.0 mm/year should be considered a maximum value for the eustatic rise in this era.

Valentin (1953) produced a chart of relative vertical movements in the British Isles from analyses of uncorrected data for 10 stations; this indicated that south of a line joining the Severn to the Thames sea level was rising at the rate of 2 mm/year. Gordon & Suthons (1963) submitted data for Newlyn, Aberdeen and Sheerness to the regression equation

$$Z_Y = aY + bP + cT, \qquad (2)$$

where P and T were the mean annual anomalies of local air pressure and air temperature respectively.

In an attempt to determine the configuration of the mean sea surface around the coastline of Europe, the author (1962) applied equation (1) to all European data for the years 1940–58. This was subsequently (1967) extended to cover all available data.

Summary of results

Various estimates of secular variations are listed in table 2. In order to support and place in context the very few values for southeast England, all British stations are included as well as those continental ones in the southern North Sea and the English Channel.

Gutenburg's results do not agree too well with those of other investigators; the data he used were generally of shorter span and, like those used by Egedal *et al.* not subjected to significant quality control.

Valentin's results are not given here, since they cover the British stations included by Egedal *et al.*, use almost exactly the same data and the same univariate analysis technique, and produce the same estimates.

For the remainder, there is reasonable agreement between independent estimates for most stations. Notable examples are Newlyn (2.3, 2.5, 2.2 mm/year), Dunbar (0.5, 0.1 mm/year) and Felixstowe (1.7, 1.6 mm/year), the three Ordnance Survey stations. Discrepancies in the estimates for Aberdeen and Sheerness by different investigators seem to arise from variations in the spans of data used. The figure of 2.8 mm/year rise at North Shields, high in comparison with those of its nearest neighbours of Dunbar (0.1 mm/year) and Felixstowe (1.6 mm/year) suggests land subsidence not unexpected in a mining area.

Particular interest has been shown in the secular changes in the Thames Estuary. Sheerness and Southend, the two major stations, are separated by only 4 km; their mean sea-level data are plotted in figure 3. The earlier Sheerness data are somewhat suspect, whereas those for Southend are considered reliable. Since about 1920 the trends at the two stations appear to be very similar, and this is borne out by the Gordon and Suthons estimate of 3.3 mm/year for Sheerness and mine of 3.4 mm/year for Southend. The earlier slower rate of rise shown graphically at Sheerness is reflected in my lower estimate of 2.4 mm/year for the span of 1874–1959. Independent analyses of annual mean high and low water heights at Southend and Tilbury for the years 1933–67 indicate trends in mean tide level of 3.4 and 2.9 mm/year respectively.

All the evidence therefore points to a relative rise in sea level in this region, over the past 50 years, of just over 3 mm/year, or 1 ft/century.

Table 2. Estimates of secular variation (mm/year) by various authors, with data years used

Bracketed figures are standard errors of estimate.
† Multivariate analyses. ‡ Incomplete data.

station	Gutenberg		Egedal *et al.*		Gordon & Suthons†		Rossiter†	
Newlyn	—		1916–1951	2.3 (0.4)	1916–1962	2.5	1915–1962	2.2 (0.1)
Avonmouth	—		1925–1950	2.4 (1.2)‡	—		—	
Liverpool	1857–1937	0.0	1858–1948	0.7 (0.3)‡	—		—	
Aberdeen	1862–1913	−0.1	1862–1913	−0.5 (0.3)	1916–1962	1.2	1874–1962	0.8 (0.1)
Dundee	—		1897–1912	−0.1 (1.4)	—		—	
Dunbar	1914–1937	−0.6	1914–1950	0.5 (0.3)	—		1914–1950	0.1 (0.2)
North Shields	—		—		—		1906–1962	2.8 (0.3)
Felixstowe	—		1918–1950	1.7 (0.3)	—		1918–1950	1.6 (0.3)
Southend	—		—		—		1929–1962	3.4 (0.4)
Sheerness	—		1835–1927	0.8 (0.1)‡	1916–1962	3.3	1874–1959	2.4 (0.2)
Delfzijl	—		1865–1950	1.2 (0.5)	—		1874–1962	1.7 (0.2)
Terschelling	—		1921–1950	1.4 (0.3)	—		1921–1962	1.8 (0.4)
Harlingen	—		1865–1950	1.7 (0.4)	—		1874–1962	1.3 (0.1)
Den Helder	—		1832–1950	1.1 (0.3)	—		1874–1962	1.4 (0.1)
Ijmuiden	—		1871–1950	0.3 (0.6)	—		1884–1962	2.4 (0.4)
Hoek v. Holland	—		1864–1950	2.6 (0.3)	—		1874–1962	2.5 (0.1)
Maasluis	—		—		—		1874–1936	1.7 (0.2)
Hellevoetsluis	—		1861–1950	1.4 (0.4)	—		1874–1962	1.7 (0.1)
Brouwershavn	—		1872–1950	1.0 (0.3)	—		1874–1962	1.5 (0.1)
Zierikzee	—		1872–1950	1.5 (0.4)	—		1874–1962	1.6 (0.2)
Vlissingen	—		1888–1950	2.6 (0.3)	—		1890–1962	3.0 (0.2)
Brest	1807–1936	0.8	—		—		1894–1961	2.1 (0.3)

Turning now to the continental results, there is excellent agreement between Brest (2.1 mm/year) and Newlyn (2.2 mm/year) on opposite sides of the English Channel; indeed, these are the only estimates available for the Channel shores. The only other evidence of any relevance to southeast England comes from the Netherlands stations. In general their data cover almost 90 years up to 1962, and suggest a relative rise in sea level of between 1.5 and 2.5 mm/year. However, it now appears that the basic data provided for the 1967 study were not always homogenous at any given station, due to undocumented datum changes; it also seems unlikely that any corrections can be incorporated. The influence of these changes is somewhat reduced, however, by restricting considerations to a 50-year period ending 1962; it then transpires that a trend of 2 mm/year is evident along the Netherlands coast.

To summarize, the most reliable values of secular variation (mm/year) during this century relevant to this discussion are as follows:

Aberdeen +0.8 and Dunbar +0.1
Felixstowe +1.6
Southend +3.4 and Sheerness +3.3
Netherlands +2.0
Newlyn +2.2 and Brest +2.1.

Taking a eustatic trend of +1 mm/year as a working hypothesis, those figures suggest a slight land uplift in eastern Scotland of the order of 0.5 mm/year, and subsidence in southern England and along the French and Dutch coasts of the order of 1 mm/year; there is a strong suggestion that this rate of subsidence is more than doubled (over 2 mm/year) in the area of the Thames.

In relative terms, i.e. irrespective of the eustatic hypothesis, there can be no doubt that Scotland is rising relative to southern England by at least 1.5 mm/year.

Future changes in mean sea level

Studies of sea-level trends are usually undertaken with a practical application in mind, and this is often to provide a forecast of probable levels. Without an adequate understanding of the processes involved, extrapolation of a well-documented trend in some ways involves acceptance of the psalmist's faith '...as it was in the beginning is now, and forever shall be...' which does not fit easily into this particular geophysical context.

When the trend is not well-documented, and this contribution outlines how little reliable knowledge is available for the British Isles, there emerges a *prima facie* case for improving the scope and quality of the basic tide gauge network. Figure 1 shows that some steps have already been taken in this direction by organizing a network of first class, orthodox tide gauges. In addition, there exists a need for a small number of 'tidal observatories' of the calibre of Newlyn, strategically situated around our coastline, at which systematic observations of winds, air pressure, water velocity, salinity and temperature are made to supplement sea level observations. If this were to be set in hand now, participants to a future Royal Society discussion on this subject would be grateful to us.

REFERENCES (Rossiter)

Colepresse, S. 1668 An account of some observations made at and nigh Plymouth, An. 1667, by way of answer to some of the quaeries concerning tides. *Phil. Trans. R. Soc. Lond.* **3**, 632–634.

Egedal, J., Disney, L. P. & Rossiter, J. R. 1954 Secular variation of sea level. *Publs scient. Ass. Océanogr. phys.* **13**, 21.

Gordon, D. L. & Suthons, C. T. 1963 Mean sea level in the British Isles. *Admty mar. Sci. Publ.* **7**, 8.

Gutenberg, B. 1941 Changes in sea level, postglacial uplift, and mobility of the earth's interior. *Bull. geol. Soc. Am.* **52**, 721–772.

Jolly, H. L. P. 1939 Supposed land subsidence in the south of England. *Geogrl J.* **93**, 408–413.

Longfield, T. E. 1932 The subsidence of London. *Prof. Pap. Ordn. Surv.* N.S. **14**.

Munk, W. & Revelle, R. 1952 On the geophysical interpretation of irregularities in the rotation of the earth. *Mon. Not. R. astr. Soc. geophys. Suppl.* **6**, 331–347.

Palmer, H. R. 1831 Description of an graphical registrer of tides and winds. *Phil. Trans. R. Soc. Lond.* **121**, 209–213.

Rossiter, J. R. 1962 Report on the reduction of sea-level observations (1940–1958) for R.E.U.N. *Trav. Ass. int. Géod.* **21**, 159–184.

Rossiter, J. R. 1967 An analysis of annual sea-level variations in European waters. *Geophys. J. R. astr. Soc.* **12**, 259–299.

Sturmy, S. 1668 An account of some observations made this present year in Hong-Road within four miles of Bristol in answer to some queries concerning the tides. *Phil. Trans. R. Soc. Lond.* **3**, 813–817.

Valentin, H. 1953 Present vertical movements of the British Isles. *Geogrl J.* **119**, 299–305.

Witting, R. 1918 Hafsytan, geoidytan och landhojningen utmed Baltiska Hafvet och vid Nordsjön. *Fennia* **39** (5), 347.